

Exercise 1

Convert each of the following IVPs in 1–8 to an equivalent Volterra integral equation:

$$y' - 4y = 0, \quad y(0) = 1$$

Solution

Let

$$y'(x) = u(x). \tag{1}$$

Integrate both sides from 0 to x .

$$\begin{aligned} \int_0^x y'(t) dt &= \int_0^x u(t) dt \\ y(x) - y(0) &= \int_0^x u(t) dt \end{aligned}$$

Substitute $y(0) = 1$ and bring it to the right side.

$$y(x) = 1 + \int_0^x u(t) dt \tag{2}$$

Substitute equations (1) and (2) into the original ODE.

$$y' - 4y = 0 \quad \rightarrow \quad u(x) - 4 \left[1 + \int_0^x u(t) dt \right] = 0$$

Therefore, the equivalent Volterra integral equation is

$$u(x) = 4 + 4 \int_0^x u(t) dt.$$